Beyond Valid Domains in Interactive Configuration

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Introduction

- *Calculation of Valid Domains* (CVD) is one of the major operations of an interactive configurator.
  - Enforces *backtrack-freeness*, and *completeness*
  - Is an *NP-hard* task
- *Compilation approach*:
  - *Offline*: all solutions compiled into a compact form (BDD, MDD, Automata, ...). Might be huge.
  - *Online*: CVD efficient (*real-time*) in the size of compiled representation
- *If we have a compact representation of all solutions, we can provide richer interaction forms than just CVD.*

Richer Interaction Forms

- Our intuition: by *visualizing* more of the *solution space structure* in each interaction step, a user can:
  - Make *more informed* navigation steps (eg. by directly observing tradeoffs between alternatives)
  - Reach its target in *less interaction* steps (eg. by fixing more variables in a single interaction step)
- A growing family of richer user interaction forms is delivered on top of compiled representations:
  - Optimal relaxations and explanations
  - Postoptimality analysis
  - Configuration with costs and preferences
- Our proposal: use *union of Cartesian products* (Cartesian Union) as a basic visualization form

CSP Formalization

Knowledge about the product (service) to be configured can be represented as a *constraint satisfaction problem* (X, D, C):

- $X = \{x_1, \ldots, x_n\}$ variables
- $D = \{D_1, \ldots, D_n\}$ domains
- $C = \{c_1, \ldots, c_m\}$ constraints
- *Sol* solution space, all valid configurations

T-Shirt Configuration

Example

Variables: *color, size* and *print*, $X = \{x_1, x_2, x_3\}$.
Domains:
$D_1 = \{black, white, red, blue\}$,
$D_2 = \{small, medium, large\}$,
$D_3 = \{MIB, STW\}$.
Rules: $F = \{f_1, f_2\}$:
$f_1 : (x_3 = MIB) \Rightarrow (x_1 = black)$
$f_2 : (x_3 = STW) \Rightarrow (x_2 \neq small)$

Figure: Valid domains

color	*size*	*print*
b,r,g,w	s,l,m	mib,stw

Figure: After size=small

color	*size*	*print*
b	s	mib

Figure: Entire solution space

color	*size*	*print*
b,r,g,w	l,m	stw
b	s,l,m	mib

Car Configuration

Example

Variables: *bumpers*,*body*,*top*,*doors*, and *hood*, $X = \{x_1, \ldots, x_5\}$.
Domains: $D_1 = \ldots = D_5 = \{white, pink, red, blue\}$.
Rules: bumpers and top should have a lighter color than body. Doors and hood must have the same color as the body.

Figure: Valid domains

bumpers	*body*	*top*	*doors*	*hood*
w,p,r	p,r,b	w,p,r	p,r,b	p,r,b

Figure: Entire solution space

bumpers	*body*	*top*	*doors*	*hood*
w	p	w	p	p
w,p	r	w,p	r	r
w,p,r	b	w,p,r	b	b

Cartesian Union as a Visualization Form

- CVD is easy to display as it is a single Cartesian product

$$VD_1 \times \ldots \times VD_n$$

- CVD is a very *coarse* visualization: $Sol \subseteq VD_1 \times \ldots \times VD_n$
- Coarse visualization leads user to make uninformed solution space navigation
- We suggest to generalize CVD so that *finer* visualizations of *Sol* are possible

Cartesian Union as a Visualization Form

- Exact *Sol* structure can be always conveyed as a union of Cartesian products:

$$Sol = \bigcup_i D_1^i \times \ldots \times D_n^i.$$

- There can be too many Cartesian products for a reasonable display of exact *Sol*
- We propose: find a Cartesian Union $\bigcup_i D_1^i \times \ldots \times D_n^i$ that strikes an adequate balance between the *preciseness of representation* and *displayability*
- We formulate a set of related problems...

Visualization Problems

Problem (Minimal Exact Representation)

For a given constraint satisfaction problem (X, D, C), with solution space Sol, what is the minimal number of Cartesian products r_{min} necessary to exactly represent Sol:

$$Sol = \bigcup_{i=1}^{r_{min}} D_1^i \times \ldots \times D_n^i.$$

Visualization Problems

Problem (Best Bounded Approximation)

For a given constraint satisfaction problem (X, D, C)*, with solution space* Sol*, and for a given maximal number of Cartesian products* r_{max} *what is the smallest over-approximation* Sol^{apx}*:*

$$Sol \subseteq Sol^{apx} = \bigcup_{i=1}^{r_{max}} D_1^i \times \ldots \times D_n^i$$

i.e. an over-approximation with the minimal number of elements?

Visualization Problems

Problem (Best Projection)

Given a solution space Sol, and a maximal number of Cartesian products r_{max} what is the subset of variables $X' \subseteq X$ yielding a Cartesian product representation

$$Sol_{X'} = \bigcup_{i} \prod_{x_j \in X'} D_j^i .$$

with at most r_{max} rows, such that projection of solution space Sol on X' variables $Sol_{X'}$, involves the maximal number of solutions?

MDD as a Compilation Structure

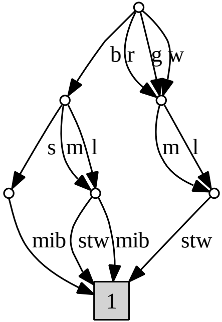
- We want a compiled representation that supports visualization forms and queries we discussed
- In this work we use Multi-Valued Decision Diagrams (MDDs), other representations might be used
- We discuss briefly some of the MDD implementation details

Multi-Valued Decision Diagrams

Definition (Multi-Valued Decision Diagram)

A multi-valued decision diagram (MDD) M is a tuple (V, r, E, var), where V is a set of vertices containing the special terminal vertex $\mathbf{1}$ and a root $r \in V$, $E \subseteq V \times V$ is a set of edges such that (V, E) forms a directed acyclic graph with r as the source and 1 as the sink for all maximal paths in the graph. Further, $var : V \rightarrow \{1, \ldots, n+1\}$ is a labeling of all nodes with a variable index such that $var(\mathbf{1}) = n + 1$. Each edge $e \in E$ is denoted with a triple (u, u', v) of its start node u, its end node u' and an associated value v.

T-Shirt MDD



color	*size*	*print*
b	s	mib
b	m,l	mib,stw
r,g,w	m,l	mib,stw

Figure: An MDD for the solution space of the T-shirt example.

MDD-Path Minimization

- Each MDD *meta-path* corresponds to a Cartesian product
- For a given variable set X, the most succinct representation corresponds to an MDD with the smallest number of *meta-paths*.
- Several methods to reduce the number of meta-paths in an MDD:
  - *Variable reordering*: the smallest MDD is not an MDD with the smallest number of paths.
  - *Non-determinization*: result is not an MDD in a strict sense
- NP-hard problem.

T-Shirt MDD Reordering

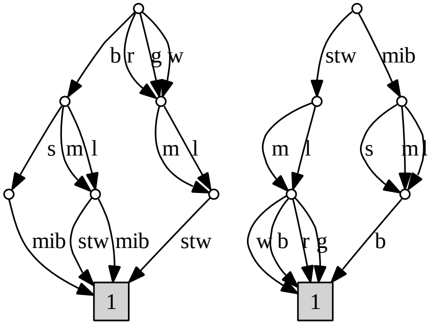


Figure: The T-Shirt example with the standard variable ordering (color, size, print) on the left, and with the new variable ordering (print, size,color) on the right. Note that the number of meta-paths reduced from three to two.

Car MDD Nondeterminization

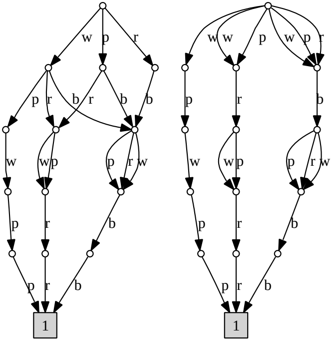


Figure: Car configuration example on the left, and after non-determinization on the right. The number of meta-paths is reduced from six to three.

Camera Catalogue

- We evaluated some of the techniques on a real-world *Camera catalogue*
- It contains 112 cameras
- Each camera has eight attributes: *brand, price, resolution, optical zoom strength, flash memory, screen size, thickness, weight*

Projection and Nondeterminization

Table: Table illustrating solution loss and row savings by projecting variables for the Camera instance.

X'	$\|Sol_{X'}\|$	P_n	P	$\|M_n\|$	$\|M\|$
1-8	112	100	106	388	394
1-7	112	94	103	317	325
1-6	112	87	99	189	200
1-5	112	83	91	143	149
2,3,4,5	111	75	92	116	121
1,2,4,5	110	61	64	79	84
1,2,3,4	109	73	78	78	80
2,4,5	108	49	54	66	65
2,3	91	32	32	34	34
2,5	88	24	24	26	26

Cameras

Price($)	Zoom	Flash (MB)
109.99+	1	16
129.99+	3	16
139.99+	3	12
149.95	3	32-
149.99	4	16
149.99+	3	16-
179.95	3-	16
179.99	3	22
199.95	3	32-
199.99	3	23-
199.99	4-	16
212.99+	6	14
215.99+	3	32
249.95	3	24-
249.99	5	24
249.99	3	32-
293.99+	10	13.4
293.99	3	16
293.99+	3.6	16
299.95+	10	32
299.95	5	32
299.95	3	256-
299.99	10-	16
299.99	5	32,8
299.99	3	32-
319.99	10	10
329.95	3	64
329.99	10	21
329.99	3	20
349.95	3	32-
349.95	1	32
349.99	12-	16
349.99	5	17
375.99	2.4	16
391.99+	12	16
399.95+	12	32
399.95	10	32-
399.95	5.8	32
399.95	3	58-
399.99	12-	16
399.99	4	32-
399.99	3	32-
401.99+	3.5	23
449.99	3	25
499.95	10.7	10
499.99	3	32-
569.99	4	32
599.99	6	32
675.99	10.7	16

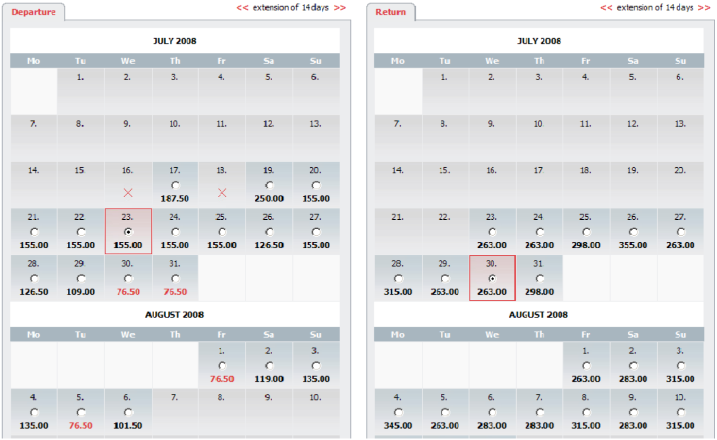
Price($)	Zoom	Flash(MB)
109.99+	1	16
129.99+	3	16
139.99+	3	12
149.95	3	32-
149.99	4	16
149.99+	3	16-
179.95	3-	16
179.99	3	22
199.95	3	32-
199.99	3	23-
199.99	1,4	16
212.99+	6	14
219.99+	3	32
249.95	3	24-
249.99	5	24
249.99	3	32-
293.99+	3.6	16
293.99+	10	13.4
293.99	3	16
299.95+	10	32
299.95	5	32
299.95	3	256-
299.99	10-	16
299.99	5	32-
299.99	3	32-
319.99	10	10
329.95	3	64
329.99	10	21
329.99	3	20
349.95	3	32-
349.95	1	32
349.99	12-	16
349.99	5	17
375.99+	2.4	16
391.99+	12	16
399.95+	12	32
399.95	10	32-
399.95	5.8	32
399.95	3	58-
399.99	12-	16
399.99	4	32-
399.99	3	32-
401.99+	3.5	23
449.99	3	25
499.95	10.7	10
499.99	3	32-
569.99	4	32
599.99	6	32
675.99	10.7	16

Are we on the right track?

- Is Cartesian Union an adequate visualization forms?
- What other forms are suitable in interactive decision making setting?
- Can we utilize existing Information Visualization techniques?
- What are the associated algorithmic challenges?

Dublin-Athens KLM

Dublin-Athens Czech Airlines



Departure << extension of 14 days >>

JULY 2008

Mo	Tu	We	Th	Fr	Sa	Su
	1.	2.	3.	4.	5.	6.
7.	8.	9.	10.	11.	12.	13.
14.	15	16. ×	17. 187.50	18. ×	19. 250.00	20. 155.00
21. 155.00	22 155.00	23. 155.00	24. 155.00	25. 155.00	26. 126.50	27. 155.00
28. 126.50	29 109.00	30. 76.50	31. 76.50			

AUGUST 2008

Mo	Tu	We	Th	Fr	Sa	Su
				1. 76.50	2. 119.00	3. 135.00
4. 135.00	5. 76.50	6. 101.50	7.	8.	9.	10.

Return << extension of 14 days >>

JULY 2008

Mo	Tu	We	Th	Fr	Sa	Su
	1.	2.	3.	4.	5.	6.
7.	8.	9.	10	11.	12.	13.
14.	15.	16.	17.	18.	19.	20.
21.	22.	23. 263.00	24 263.00	25. 298.00	26. 355.00	27. 263.00
28. 315.00	29. 263.00	30. 263.00	31 298.00			

AUGUST 2008

Mo	Tu	We	Th	Fr	Sa	Su
				1. 263.00	2. 283.00	3. 315.00
4. 345.00	5. 263.00	6. 283.00	7. 283.00	8. 315.00	9. 283.00	10. 315.00

Dublin-Athens Aer Lingus

Select Dates

From Dublin To Athens

Jul/Aug 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		16	17	18 €309	19	20 €219
21 €150	22	23 €150	24	25 €249	26	27
28 €209	29	30 €189	31	01 €160	02	03 €136
04 €102	05	06 €69				

Sold out
No flight on this day

From Athens To Dublin

Jul/Aug 2008

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		16	17	18 €160	19	20 €136
21 €126	22	23 €126	24	25 €309	26	27
28 €209	29	30 €189	31	01 €160	02	03 €160
04 €189	05	06 €189	07	08 €136	09	10 €160
11 €189	12	13 €150				

Sold out
No flight on this day

Cameras - revisited

Flash (MB)

Zoom	10	12-14	16-17	21-24	32	58	64	256
1			110		350			
3	150	140	130	180	150	400	330	300
4			150		400			
5	400		350	250	300			
6		213	400		400			
10	320	294	300	330	300			
10.7	500		676		400			
12			350		400			

Summary

- We identified an opportunity of enhancing CVD when compiled representation accessible
- We suggested a *union of Cartesian products* as a visualization form, and identified several problems to finding good Cartesian unions
- We implemented some of techniques based on MDD representation of solution space, and evaluated them on a real-world Camera catalogue
- In future: explore adequacy and algorithmic challenges for other visualization forms